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MEMORANDUM

DATE: April 27, 2008

TO: LIGO Charging Group
FROM: Rupal S. Amin LSU/LLO
SUBJECT: Electron bare mass measurement using gravitational-wave detectors
Refer to: LIGO- T070281-00-R

Abstract:
A paper by LSU's R. O'Connell was identified to LIGO charging group. Although the title connotes noise source identification due to static charges, the emphasis is heavily on using gravitational-wave detectors to measure the bare mass of electrons via test mass displacement fluctuations. I show below that these fluctuations may not be approachable using practical technology.

Section 1
During the month of October 2007, a paper was circulated throughout the LIGO charging group regarding charge effects on gravitational-wave detectors. This paper, published in Physics Letters A addresses both bar and interferometeric detectors and calculates the coordinate power spectrum due to static charges in two types of gravitational-wave detectors. Unfortunately evaluating the terminal result implies that LIGO-type detectors would need to be sensitive to displacements at many orders of magnitude below the initial LIGO SRD [1].

Section 2
The displacement fluctuation model begins with a general dissipative scheme. The dissipation model uses a generalized quantum Langevin equation the general dissipative model.

$$m\ddot{x} + \int_{-\infty}^{t} \mu(t-t')\dot{x}(t')dt + V'(x) = F(t) + f(t).$$

Here the μ(t) is the memory function; the V'(x) is dV(x)/dx; F(t) is a random force or the noise force; f(t) is a c-number force. This is a central equation that shows up in many of O'Connell's papers starting in 1988 [2]. The following equation informs readers that the independent oscillator Hamiltonian will be activated paired with a generic heat bath. These heat baths are later specified as an Ohmic and radiation bath.

The derivation of the displacement power spectrum, equation (11) is obtained by assuming that the memory function can be fashioned from a large number of oscillators each affected by a Heavyside step function. Combined with the autocorrelation function the displacement power spectrum of the coordinate fluctuations is

$$P(\omega) = \frac{\hbar}{\pi} \frac{\omega Re\left(\tilde{\mu}(\omega)\right) \coth\left({}^{\hbar\omega}/_{2kT}\right)}{\left\{m^2 \left(\omega^2 - \omega_0{}^2 - \frac{\omega}{m} Im(\tilde{\mu}(\omega)\right)^2 + \left(\omega Re\tilde{\mu}(\omega)\right)^2\right\}}.$$

The integral of this power spectrum is the mean square displacement which leads us to RMS displacement.

$$\langle x^2 \rangle = C(0) = \int_0^\infty d\omega\, P(\omega)$$

Section 3

O'Connell next discusses the consequences of the dissipation equations for a Ohmic heat bath, a heat bath for interferometers, and a radiation heat bath for oscillatory detectors. The Ohmic heat bath is conventionally used to describe the cryogenic bar detectors. The resulting mean square position is correct at the low temperature limit where bar detectors operate. This is due to the resonant integrand that models the narrow bandwidth based on the quality factor of such a device.

The second heat bath discussion discusses the interaction for non-resonant interferometeric detectors such as LIGO. O'Connell states previous theories based on the (phenomenological, complex spring based) Zener function are not entirely helpful. Rather fits to the Fourier transform of the memory function have several advantages since it is only dependent on the heat bath parameters regardless of the nature of the heat bath.

Section 5 of O'Connell's paper is a more extended discussion since examples are given assuming a stray charge affects a bar detector using the memory function. Also, a relation between the renormalized mass and the electron bare mass is written. A short calculation reproduded below notes how the mean square displacement is affected by the appearance of the bare electron mass. Finally, a brief calculation using the LSU cryogenic bar's parameters is used. I have extended this calculation to use the LIGO mirrors. Unfortunately, you will read in the following section that even LIGO cannot measure the electron bare charge.

Section 4

As mentioned before, the radiation bath is more extended. The only assumption needed is the form factor of the charge distribution. Albeit, this choice did not greatly affect the final result Equation (19) lays out the structure of μ's Fourier transform where capital M is the renormalized mass of the electron bare mass. Combining equation (19) with the power spectrum equation, the solution to the radiation bath is shown for completeness below.

$$\langle x^2 \rangle = \frac{\hbar \tau_q}{\pi M} \log\left(\frac{M}{m\omega_0 \tau_q}\right)$$

Where $\tau_q = \frac{2}{3} \frac{q^2}{M} \frac{1}{c^3}$

q is the amount of charge. c is the speed of light.

The log term here does not dominate unless m goes to zero. For LIGO and ALLEGRO, the multiplier is the important part.

Assume that 10^{-9} C of charge builds up on ALLEGRO's resonant bar and that τ_q is 10^-37 s, the RMS displacement is 10^-36 m! Let's go to a LIGO IFO. Assume that the test masses are 10 kg, and the same amount of charge builds up 10^{-9} C. This yields a τ_q of 10^{-35} s. If I stuff this into the logarithm's multiplier, I get and RMS displacement of 10^{-34} m!!! This is one order of magnitude above Planck length.

Our only hope is to reduce the mass of the mirrors to that of an electron or "reasonable" size atom. Doing this first raises τ_q to 10^{-24} s. The resulting RMS displacement is roughly 10^{-14} m. But naturally, there is no way we can bounce 2 MW of laser light off one electron.

The other result in O'Connell's paper is that for displacement mean square the high temperature limit (thermally driven noise) exceeds the low temperature limit. The low temperature limit then far exceeds the displacement mean noise of a few charges sitting on the test mass—assuming that the bare mass is not zero.

References:

1) LIGO-E950018-02 E

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