

# Scaling law in signal-recycled interferometers

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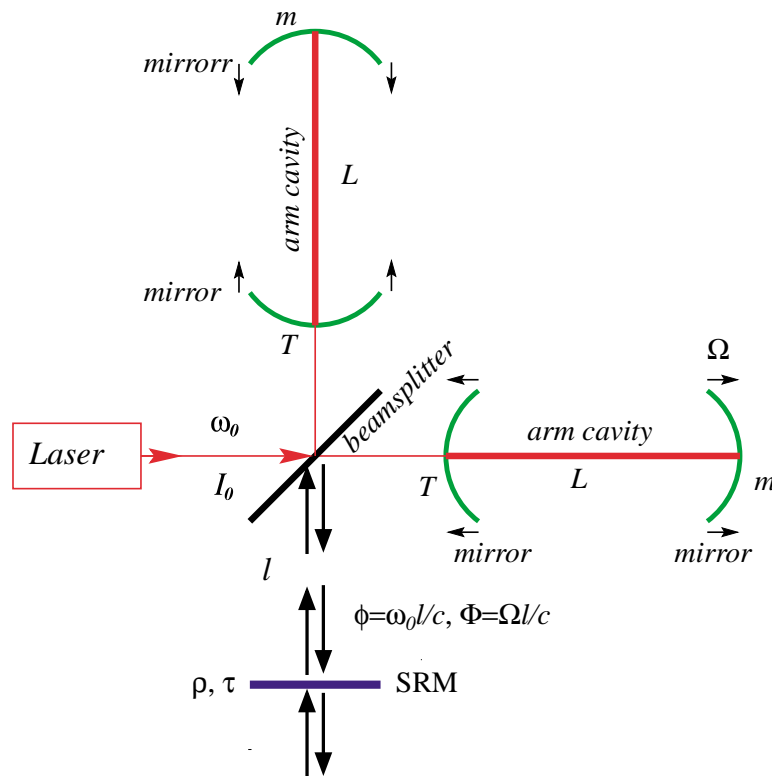
*Based on:*

Buonanno and Chen, [gr-qc/0208048](#)

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## Signal-recycled interferometer of Advanced LIGO



The Presence of a Signal-Recycling Mirror (SRM)

[At all optical powers]

- Modifies optical resonant frequency and storage time. [Drever, 82; Meers, 88; Mizuno, 95.]

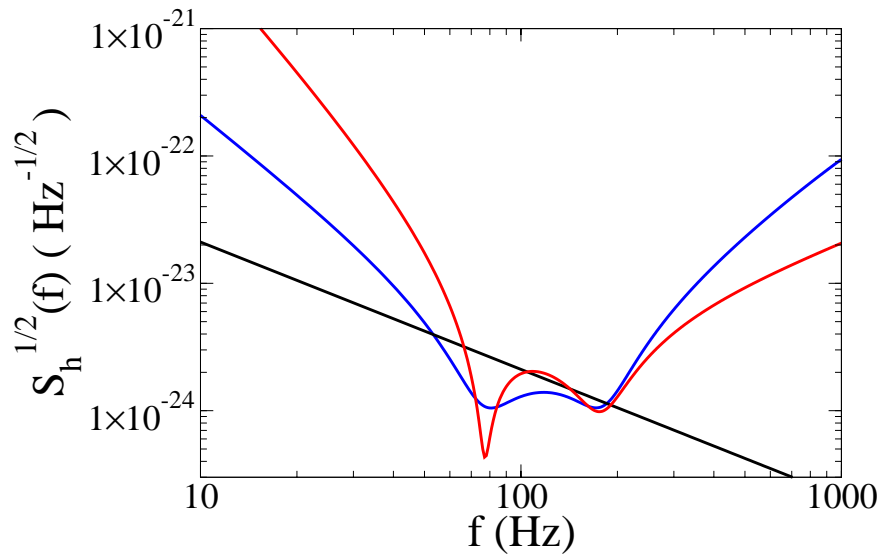
[At high enough optical power, e.g., in LIGO-II]

- Creates optomechanical coupling that changes test-mass dynamics. [Buonanno and Chen 01-02]
- Allows the interferometer to beat the SQL [Buonanno and Chen 01-02]

Conv (basic)  $\omega_0, I_0, L, T, m$   
 SR (extended)  $\rho, \tau, l, \phi = [\omega_0 l / c]_{\text{mod } 2\pi}, \Phi \equiv \Omega l / c \rightarrow 0$

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**Exploring the parameter space: *scaling law?***  
**[Buonanno and Chen 01–02, Fritschel, Shoemaker, Strain . . . ]**

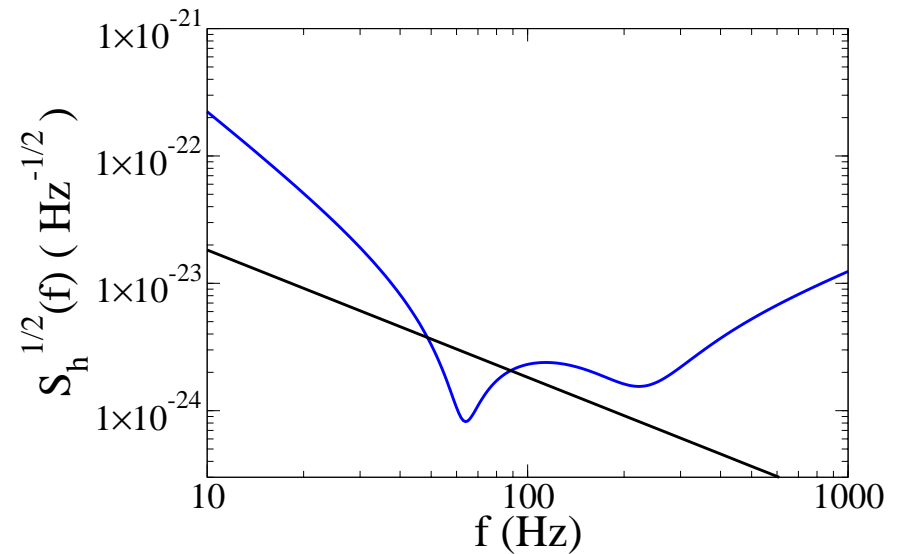


Example given by Buonanno and Chen:

$$I_{BS} = 10 \text{ kW}$$

$$T = 0.033, \gamma_{\text{arm}} = 2\pi \times 100 \text{ Hz}$$

$$\rho = 0.9, \phi = \pi/2 - 0.47$$



Reference design by LIGO experimentalists:

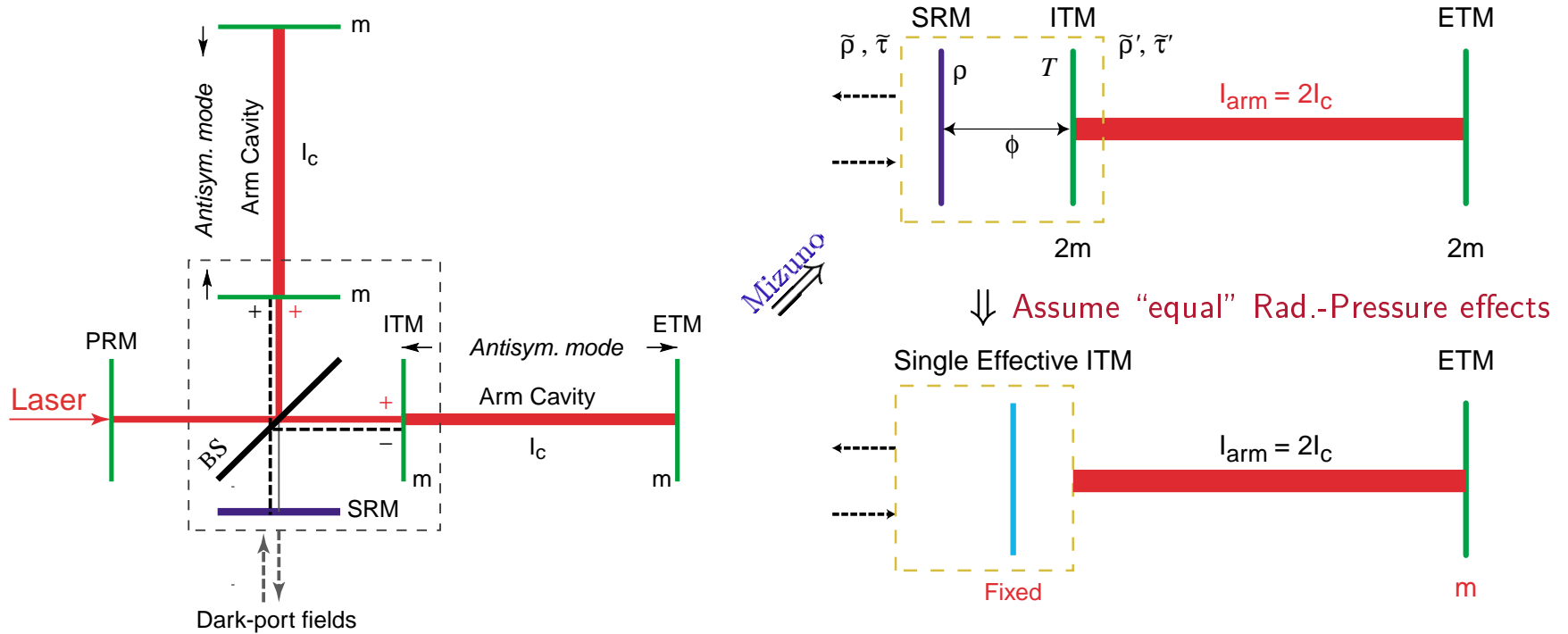
$$I_{BS} = 2.5 \text{ kW}$$

$$T = 0.005, \gamma_{\text{arm}} = 2\pi \times 15 \text{ Hz}$$

$$\rho = 0.964, \phi = \pi/2 - 0.06$$

# Mapping SR interferometer to single-detuned cavity

[Mizuno, 95; Rakhmanov, 00; Khalili, 01; Buonanno and Chen, 02]



Optics: *well known*,  $(T, \rho, \phi) \rightarrow \tilde{\rho}' \rightarrow \omega_{\text{opt res}} = \omega_0 - \lambda - i\epsilon$   
 Opto-Mechanics: *almost trivial*, [Approximation made, though]

$$I_c, m, L \rightarrow \nu_c \equiv \frac{8\omega_0 I_c}{m L c} \sim \Omega_{\text{GW}}^3$$

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## The use of the characteristic description

[Buonanno and Chen, 02]

- Previous results of SR interferometers at high powers [Buonanno and Chen, 00–01] written in characteristic parameters: equations much simpler, more physical. For example, the *optical spring* constant:

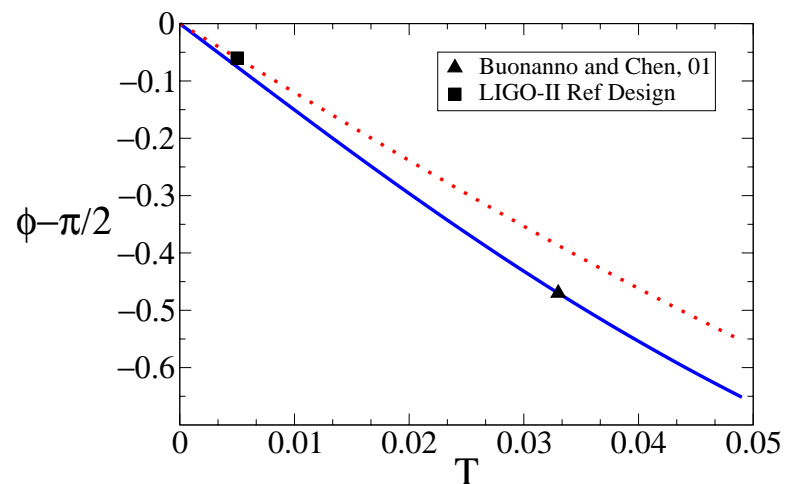
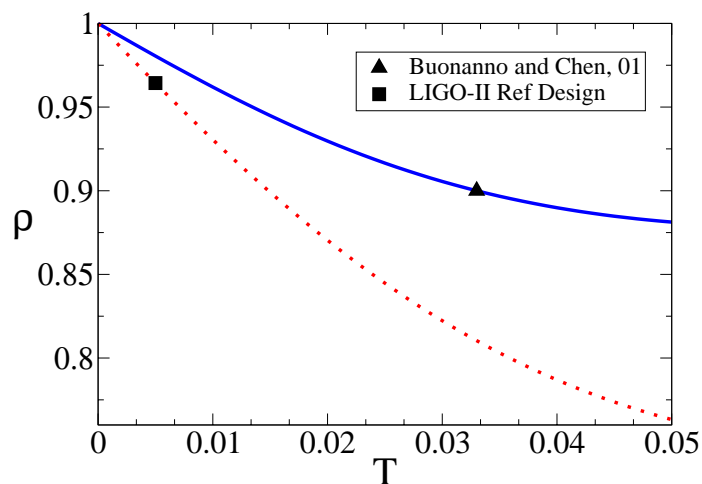
$$\frac{K_{\text{opt}}(\Omega)}{m/4} = \frac{-\lambda \iota_c}{(\Omega - \lambda + i\epsilon)(\Omega + \lambda + i\epsilon)} \dots = \Omega^2$$

See Buonanno and Chen, gr-qc/0208048 for more nice results.

- Scaling laws:
  - **Optical**, confirmed that, with  $I_c$ ,  $m$ ,  $L$  fixed, scaling laws in the low power regime still valid here:  $(\lambda, \epsilon) = \text{const}$ . Can be used for LIGO-II optimization.
  - **Opto-Mechanical**, one more scaling relation:  $\iota_c \equiv \frac{8\Omega_0 I_c}{mLc} = \text{const}$ . Relating LIGO-II to experiments in other regimes.

## Using the Optical scaling law: simplifying LIGO-II optimization

For configurations with the same  $I_c$ ,  $m$  and  $L$ , fix  $\lambda$  and  $\epsilon$ , and:



A:  $(\lambda, \epsilon) = 2\pi \times (195 \text{ Hz}, 25 \text{ Hz})$ , contains  $(T, \rho, \phi) = (0.033, 0.9, \pi/2 - 0.47)$ . [Buonanno and Chen, 01-02]

B:  $(\lambda, \epsilon) = 2\pi \times (228 \text{ Hz}, 69 \text{ Hz})$ , contains  $(T, \rho, \phi) = (0.005, 0.964, \pi/2 - 0.06)$ . [LIGO-II Reference Design.]

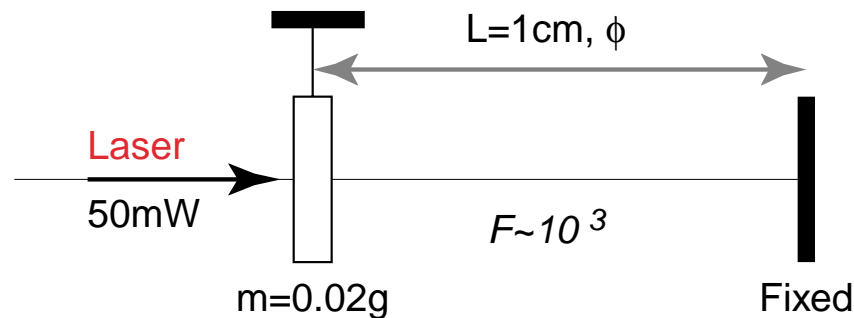
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## Using the full Optomechanical scaling law

### Linking LIGO-II and table-top experiments

Table-top experiment proposed by Braginsky, Khalili and Volikov (2001), single detuned cavity with one movable mirror. [Directly analyzed by Khalili, (2001)]

. . . In a very different regime from LIGO



now fits into the same formalism as LIGO-II:

$$\left| \begin{array}{c|c|c} \epsilon \sim \lambda/\sqrt{3} & \nu_c^{1/3} & \Omega \\ \hline 5 \times 10^7 \text{ s}^{-1} & 10^5 \text{ s}^{-1} & 10^4 \text{ s}^{-1} \end{array} \right|$$

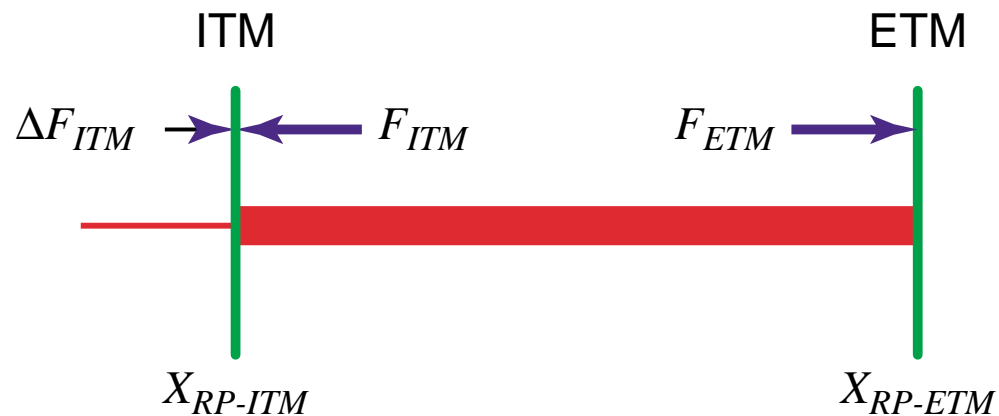
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## Approximations made in the quantum optical analysis of LIGO interferometers

So far: leading order in  $T$  and  $\Omega L/c$ .

Errors associated:

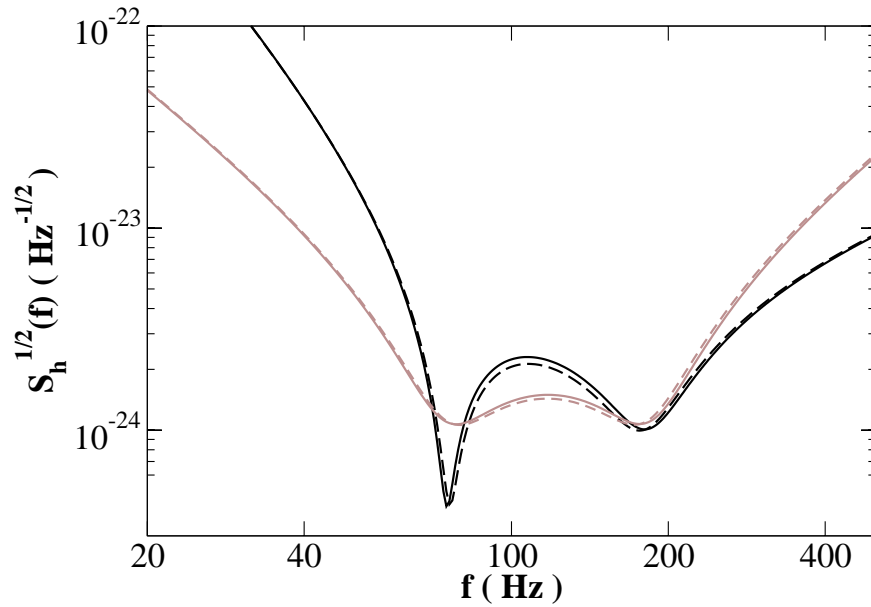
- **Difference** in the ITM and ETM contributions to radiation-pressure effects **ignored**. *Still to be analyzed.*



- Propagations of optical fields less accurate. *Resolved by this work.*

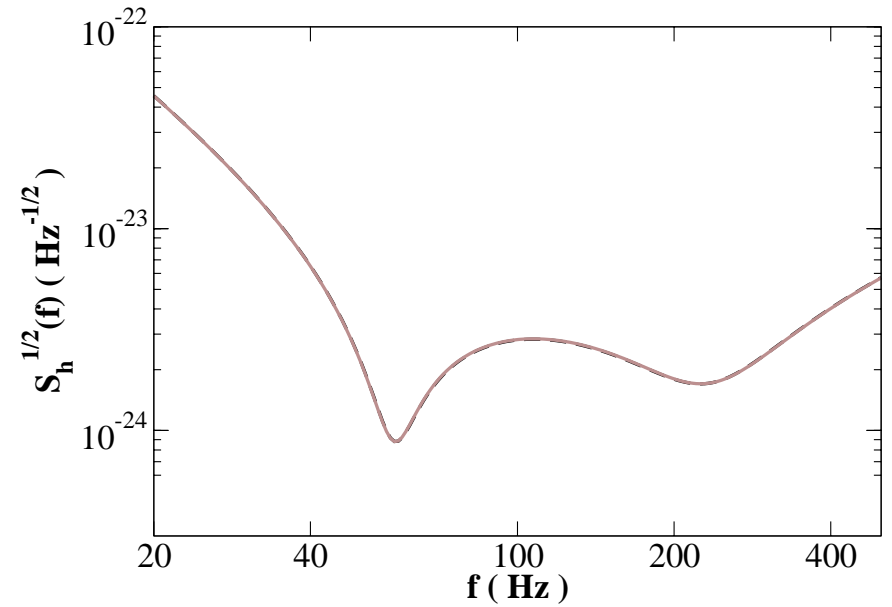
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## Leading-order in $T$ vs all orders in $T$



Example given by Buonanno and Chen:

$$T = 0.033, \rho = 0.9, \phi = \pi/2 - 0.47$$



LIGO-II Reference design:

$$T = 0.005, \rho = 0.964, \phi = \pi/2 - 0.06$$

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## Summary

- SR interferometer *with high laser power* (with very short SR cavity) mapped into a single detuned cavity
- Scaling law among experimental parameters
- Scaling law helps:
  - presenting calculation results in a simpler way, more physical (!)
  - optimization for LIGO-II
  - relating LIGO-II to table-top experiments
- Remaining issue: difference in the contributions of ITM and ETM to radiation-pressure effects.