



LIGO I: Bounding the Possible

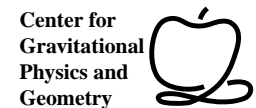
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LIGO Scientific Collaboration Meeting

LIGO-G000291-00-D





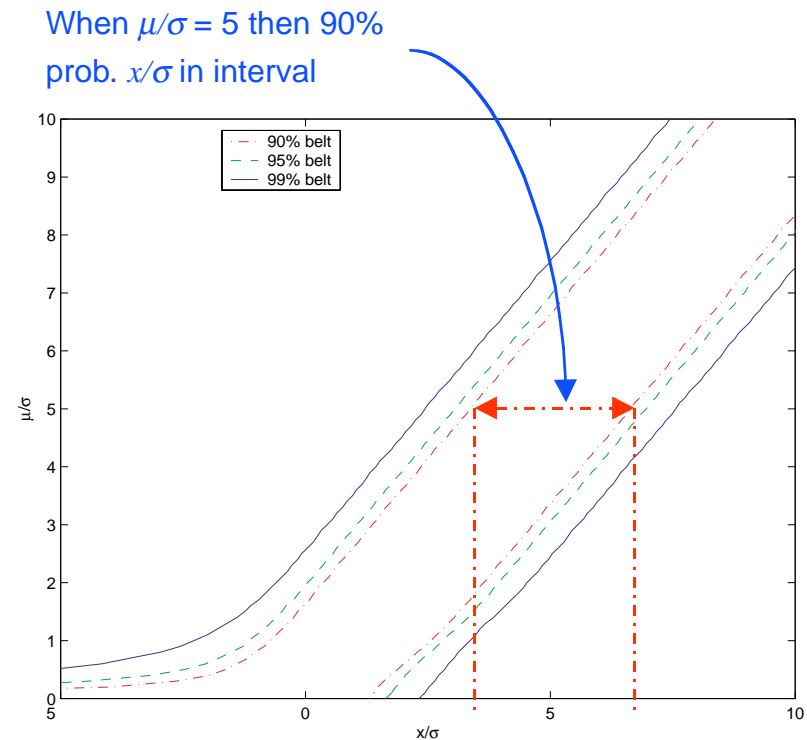
Bounding the Possible: Upper Limits

- Source strength
 - » Stochastic signal
 - » Pulsars & all-sky, all frequency CW search
- Population
 - » Limit average wave strength over population: e.g., γ -ray bursts, supernovae
- Event rates
 - » Unanticipated burst sources
 - Waveform unknown
 - » Anticipated burst sources
 - Waveform known, unknown
- Outline
 - » Confidence intervals, upper limits & credible sets
 - » Stochastic Signals
 - » Pulsars
 - » Burst sources
- Assumptions
 - » Best estimate $h(t)$ has all identifiable instrumental signatures removed or vetoed
 - Relaxing assumption *ad hoc* until actual data is available: each can make their own from estimates here



Confidence Intervals and Upper Limits

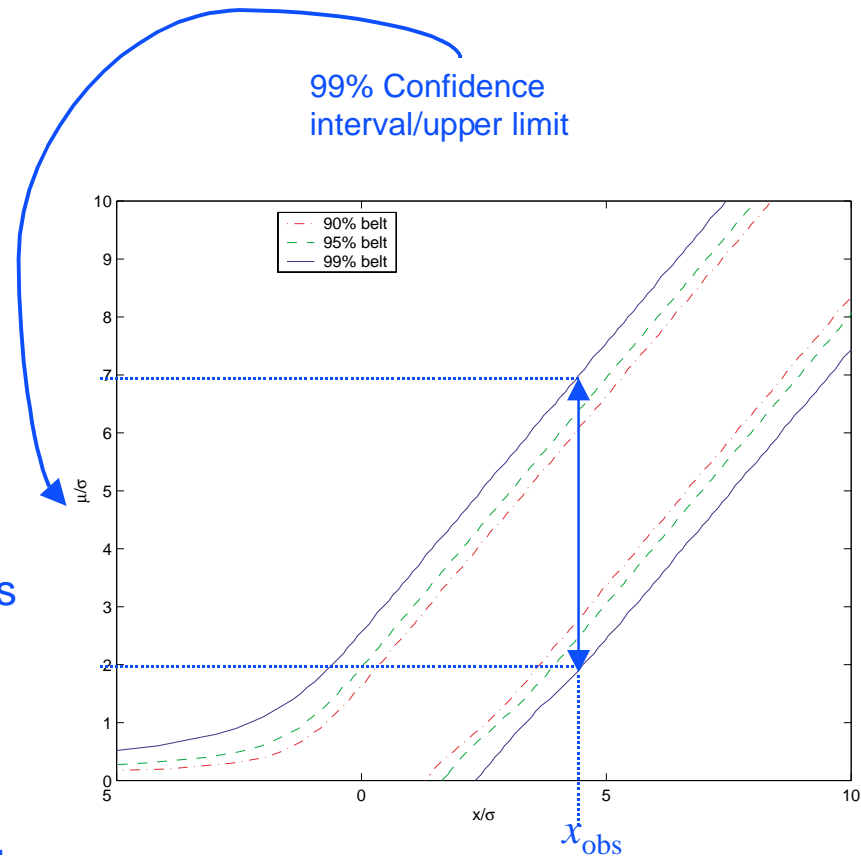
- Meaning
 - » Range of hypotheses μ for which prob. of observation x is p
- Choices
 - » Defining range: e.g.,
 - Smallest interval: $P(x/\mu) > K$
 - Lower (upper) limit: $P(x < x_0/\mu)$ [or $P(x > x_0/\mu)$]
 - “Unified”: $P(x/\mu)/P(x/\mu_{best}) > K$
 - » Different choices, different intervals
- Confidence Belt
 - » Range of probable observations as function of hypothesis





Confidence Intervals and Upper Limits

- Meaning
 - » Range of hypotheses θ for which prob. of observed x is p
- Choices
 - » Defining range
 - » Different choices, different intervals
- Confidence Belt
 - » Range of probable observations as function of hypothesis
- Interval
 - » Range μ for which x_{obs} likely
- Ref.: Feldman & Cousins, Phys. Rev. D., **57:7**, 3873 (1998)





GW From Pulsars: Observational Limits, Theoretical Prejudices

- Wave amplitude (rms & angle-averaged):

- »
$$h_{rms} = 10^{-26} \left(\frac{10 \text{ Kpc}}{r} \right) \left(\frac{\varepsilon}{10^{-6}} \right) \left(\frac{I}{10^{45} \text{ g cm}^{-3}} \right) \left(\frac{f}{100 \text{ Hz}} \right)^2$$

- Observational Limits

- » Pulsar timing of spindown shows dominant spin dissipation mechanism can't be GW
 - $L \sim f^n$; $n_{\text{obs}} \sim 4 - 5$, $n_{\text{GW}} > 6$
- » GW luminosity must be much less than implied by spindown
 - $\varepsilon < 10^{-8}$ for ms (recycled) pulsars

- Theoretical Prejudices

- » Radiation associated with non-axisymmetric deformation of crust; crust material strength limits ε less than several 10^{-6}



GW From Pulsars: Setting Upper Limits

- Beat output against known phase

$$\rho = \left| \int_0^T dt h(t) e^{-i\Phi(t)} \right|$$

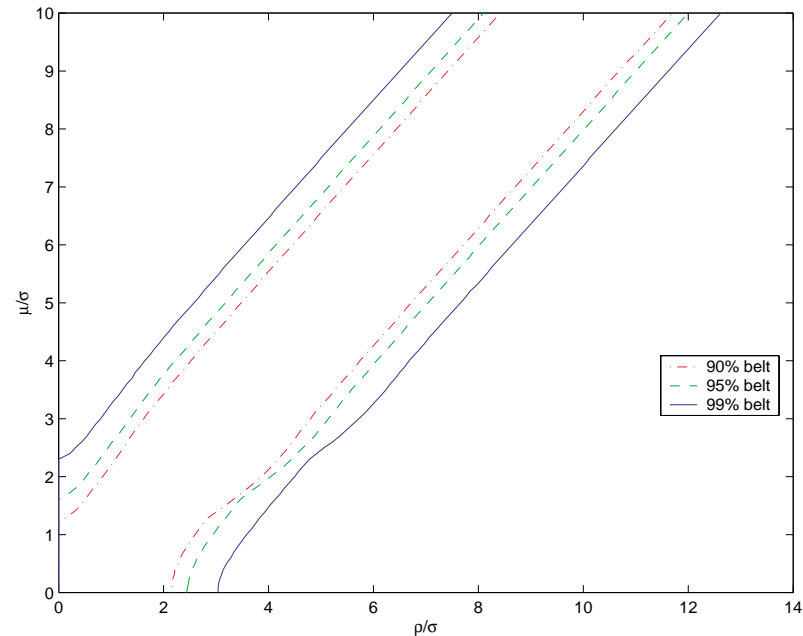
- Statistic: $|\rho|$

- » Sampling distribution: Rice

- $P(\rho | \mu) = \frac{\rho}{\sigma^2} \exp\left[-\frac{\rho^2 + \mu^2}{2\sigma^2}\right] I_0\left(\frac{\rho\mu}{\sigma^2}\right)$
- Variance σ^2 : ms noise in band
1/T about f_{GW}

- For 1 year integrated observation ...

- » LIGO I sets limit $\varepsilon_{99\%} \sim 2 \times 10^{-6}$ on
50 Hz pulsar at 10 Kpc





Unanticipated Transients: External Trigger

- GW Bursts correlated with other observational channels (e.g., EM, neutrinos)
 - » Think SN, γ -ray bursts
- Test statistic: integrated cross-correlation of GW detectors
 - » $c = \iint dt dt' h_1(t)h_2(t')Q(t-t')$
 - » About, , in direction of, burst: on-source
 - » Other times: off-source
 - » Compare population means
 - t statistic
 - » Limit on *in band population mean* h^2
 - $h < 10^{-21}$ (in band; 99%) with 1000 observed γ -ray events and broadband GW-burst model



Unanticipated Transients: GW-Only

- New feature: internal event trigger
 - » Trigger distinguishes on, off source
 - » Possible triggers:
 - “Energy” threshold
 - Event classification (energy spectrum, time-frequency power distribution, etc.)
- Correlate between detectors on, off source
 - » $c = \iint dt dt' h_1(t)h_2(t')Q(t-t')$
 - » Use t -statistic to compare on, off source populations
- Weaker limit than with astrophysical trigger
 - » Unknown source direction, trigger efficiency weakens limit
 - » Background exclusion becomes critical



Summary

- LIGO I will set physically significant upper limits
 - » Upper limits all improve with observation time
 - » LIGO engineering run starts the ball rolling!
- Pulsars
 - » $\varepsilon_{99\%} \sim 10^{-4}$ in one week run at 1/10 sensitivity!
 - » Test theoretical prejudice based on crust strength upper limit
- Unanticipated GW bursts
 - » Model dependent, but $h_{UL} \sim 10^{-21}$ improves bar limits by factor 100 over broader band
- Supernovae
 - » Opportunistic constraint on SN efficiency
 - » Luck favors the prepared mind ...